Wind Power

The SIROCCO Project is a fictitious wind turbine installation, slated to be built on the Adriatic coast of Croatia.

Assumptions and Givens

The formula for the circumference of a circle is $C = 2\pi r$, where *r* is the radius. The formula for the area of a circle is $A = \pi r^2$.

Question #11:

The SIROCCO Project is looking at different models of wind turbines to use for its wind farm on the Croatian coast. One of the factors that is important to the developers is the length of the blades. The model they are considering—the CA1 Wyndmill—has a blade length of 37.5 meters. What distance will the tip of this blade travel as it completes one full revolution?

a. 117.8 m
b. 178.5 m
c. 225.0 m
d. 235.6 m
e. 252.4 m

Solution: As the blades of the wind turbine spin, they trace out a circle with a radius of 37.5 meters. The total distance traveled by the tip of one of the blades is just the circumference of this circle. Using the given expression for the circumference of a circle, we find that the blade tips travel $C = 2\pi\ (37.5\text{ m}) = 235.6\text{ m}$. The correct answer is d.

Assumptions and Givens

The formula for speed is $s = \frac{d}{t}$, where *d* is the distance traveled and *t* is time.

Question #12:

The manufacturer of the CA1 Wyndmill (the same turbine as in Question #1) informs the developers that, when working at maximum efficiency, the blades complete 900 full revolutions every hour. In this case, how fast (in meters per second) do these blades spin?

a. 51.2 m/s
b. 58.9 m/s
c. 62.7 m/s
d. 68.1 m/s
e. 72.2 m/s

Solution: From Question #11, we know that the blades travel 235.6 meters during one full revolution. If the blades complete 900 revolutions every hour, then they travel $\frac{900\text{ rev}}{\text{hour}} \times \frac{235.6\text{ m}}{\text{rev}} = \frac{212040\text{ m}}{\text{hour}}$. This is the speed in meters/hour. To find the speed in meters/second, we can just divide this by 3600 (the number of seconds in an hour): $\frac{212040\text{ m}}{\text{hour}} \times \frac{1\text{ hour}}{3600\text{ s}} = \frac{58.9\text{ m}}{\text{s}}$. The correct answer is b.

Question #13:

Croatia has a long coastline on the Adriatic Sea, which is why it's such a perfect spot for a wind power project. However, the Adriatic Sea has different wind currents that occur at different times of the year. (The SIROCCO Project gets its name from one of these winds: the sirocco.) The table below shows the average wind speed on the coast during each month of the year.

Month	Average Wind Speed (m/s)
January	3.5
February	3.8
March	4.2
April	5.1
May	4.4
June	4.2
July	3.9
August	3.2
September	3.1
October	3.8
November	3.9
December	3.7

If the developers give each month equal weight, what is the average wind speed for the entire year?

a. 3.3 m/s
b. 3.6 m/s
c. 3.9 m/s
d. 4.2 m/s
e. 4.5 m/s

Solution: To find the average wind speed for the entire year, we can take the average of the monthly data given in the data table. Adding up all of the values in the table, we get 46.8, and there are 12 months in the year, so: $\frac{46.8}{12} = 3.9$ m/s. The correct answer is c.

Assumptions and Givens

The formula for the theoretical maximum power that can be produced by a wind turbine is $P_{\text{max}} = \frac{1}{2}\rho A v^3$, where ρ is the air density, *A* is the area swept out by the blades of the turbine, and *v* is the wind speed. Power is measured in watts (W), with $1\ \text{W} = 1\ \frac{\text{kg m}^2}{\text{s}^3}$.

Question #14:

The SIROCCO developers have decided to use the CA1 Wyndmill. Before they design their wind farm on the coast, they want to know the maximum possible power that a single wind turbine could generate in this environment. The air pressure at sea level on the Adriatic coast is 1.23 kg/m^3, and the average wind speed is 3.5 m/s. If the length of the blades is 37.5 m, what is the maximum power that this turbine can produce?

a. 79,521 W
b. 87,698 W
c. 95,121 W
d. 103,584 W
e. 116,491 W

Solution: Before using the given formula for the maximum power, we need to calculate the area swept out by the blades, *A*. The area of a circle is πr^2, and for this turbine $r = 37.5$ m. Thus, $A = \pi(37.5\ \text{m})^2 = 4417.86\ \text{m}^2$. Now we plug this value and the given values for the air density and wind speed into the formula for the maximum power: $P_{\text{max}} = \frac{1}{2}\left(1.23\ \frac{\text{kg}}{\text{m}^3}\right)(4417.86\ \text{m}^2)\left(3.5\ \frac{\text{m}}{\text{s}}\right)^3 = 116{,}491$ W. The correct answer is e.

Assumptions and Givens

The percentage power loss in a turbine, C_t, is the power lost due to mechanical inefficiencies. The value of C_t can fall between 0 and 1, with 0 meaning all the power is lost and 1 meaning no power is lost. Thus, the actual maximum power that a turbine can generate, P_{actual}, can be found by using the formula $P_{\text{actual}} = P_{\text{max}} \times C_t$.

Question #15:

Despite the developers' hopes, wind turbines never produce the maximum amount of power possible. There is always some power loss in any mechanical system. This loss is caused by friction between the moving parts of the turbine, and energy escapes the system as noise and heat. In an attempt to quantify this loss, the developers install one turbine and run a test to compare its theoretical power production with its actual power production. Given the conditions on that day, the turbine should have generated 105,000 W, but the maximum power it actually generated was 33,600 W. What is the power loss for this turbine?

a. 0.32
b. 0.37
c. 0.43
d. 0.46
e. 0.49

Solution: The problem provides us with values for P_{max} and P_{actual},so we need to rearrange the equation to solve for C_t: $C_t = \frac{P_{\text{actual}}}{P_{\text{max}}}$. Then we can plug in the values to solve the equation: $C_t = \frac{33600\text{ W}}{105000\text{ W}} = 0.32$. The correct answer is a.

Assumptions and Givens

1 acre is equivalent to 0.004 km^2.

Question #16:

The developers finally decide to place the order for the wind turbines. However, they need to determine how many turbines they can fit on the strip of coastal land allotted for the SIROCCO Project, which measures 54 km^2. The manufacturers of the wind turbines say that each turbine needs 64 acres of land to be installed and function properly (the turbine's "footprint"). What is the maximum number of turbines that the developers can fit on their piece of land?

a. 121
b. 210
c. 393
d. 428
e. 605

Solution: The area of the project's land is given in km^2, but the turbine footprint of given in acres. So, we need to convert the land area from km^2 to acres: $54\ \text{km}^2 \times \frac{1\ \text{acre}}{0.004\ \text{km}^2} = 13{,}500$ acres. This is the area of the project land in acres. Now, we can divide by the footprint of one turbine to find how many turbines could (theoretically) fit on this land: $13{,}500\ \text{acres} \times \frac{1\ \text{turbine}}{64\ \text{acres}} = 210.94$. Since we can't have a fraction of a turbine, the maximum number of turbines that can fit is 210. The correct answer is b.

Question #17:

After reviewing the specific layout of the project's land, the developers decide to install 95 wind turbines. They would like an estimate of the *actual* maximum power output from the turbines. Because SIROCCO is a cautious organization, they want to be conservative with their estimate; thus, they make the following assumptions about the conditions on their wind farm and the functioning of the turbines:

$$\boldsymbol{\rho = 1.23\ \frac{kg}{m^3}}$$
$$\boldsymbol{v = 2.5\ \frac{m}{s}}$$
$$\boldsymbol{r = 37.5\ m}$$
$$\boldsymbol{C_t = 0.45}$$

Given these values, what is the total *actual* maximum power that these 95 turbines could generate?

a. 952,640 W

b. 1,102,910 W
c. 1,396,890 W
d. 1,728,440 W
e. 1,814,880 W

Solution: To solve this problem we need to make use of two equations: 1) the equation for P_{max}, and then 2) the equation for P_{actual}. First, we can calculate P_{max} for one of these turbines under the given conditions: $P_{\text{max}} = \frac{1}{2}\rho A v^3 = \frac{1}{2}\left(1.23\,\frac{\text{kg}}{\text{m}^3}\right)\left[\pi(37.5\text{ m})^2\right]\left(2.5\,\frac{\text{m}}{\text{s}}\right)^3 = 42{,}453\text{ W}$. Next, we can calculate P_{actual} for one of the turbines: $P_{\text{actual}} = P_{\text{max}} \times C_t = 42{,}453\text{ W} \times 0.45 = 19{,}104\text{ W}$. This is the actual maximum power output for one turbine. Finally, to find the total *actual* maximum power for the whole wind farm, we can multiply this result by 95: $19{,}104\text{ W} \times 95 = 1{,}814{,}880\text{ W}$. The correct answer is e.

Assumptions and Givens

The formula for percentage change is $\frac{\text{new value - old value}}{\text{old value}} \times 100\%$.

Question #18:

While installing the other turbines, the developers continue to run tests and collect data using the first turbine. One day, a powerful storm rolls in and brings strong winds. Before and during the storm, they collect the following air density and wind speed data:

	ρ	*v*
Before storm	$1.23\ \frac{\text{kg}}{\text{m}^3}$	3.4 m/s
During storm	$1.02\ \frac{\text{kg}}{\text{m}^3}$	9.6 m/s

What is the percentage change in the theoretical maximum power (P_{max}) of the wind turbine? (Again, the blade length of the Wyndmill model is $r = 37.5$ m.)

a. 100%
b. 304%
c. 844%
d. 1,253%
e. 1,767%

Solution: The first step is to calculate P_{max} both before and during the storm. Before the storm, we have $P_{\text{max}} = \frac{1}{2}\rho A v^3 = \frac{1}{2}\left(1.23\,\frac{\text{kg}}{\text{m}^3}\right)\left[\pi(37.5\text{ m})^2\right]\left(3.4\,\frac{\text{m}}{\text{s}}\right)^3 = 106{,}788$ W. During the storm, we have $P_{\text{max}} = \frac{1}{2}\rho A v^3 = \frac{1}{2}\left(1.02\,\frac{\text{kg}}{\text{m}^3}\right)\left[\pi(37.5\text{ m})^2\right]\left(9.6\,\frac{\text{m}}{\text{s}}\right)^3 = 1{,}993{,}408$ W. Then, to find the percentage change, we can plug these values into the given equation: $\frac{1{,}993{,}408\text{ W} - 106{,}788\text{ W}}{106{,}788\text{ W}} \times 100\% = 1{,}767\%$. The correct answer is e.

Question #19:

All of the wind turbines have been constructed, and the SIROCCO Project is ready to launch. Now, they are interested in learning about the return on their investment. The total cost of installing the SIROCCO Project was €485,000,000. However, after studying energy prices and usage in Croatia, they believe that the wind farm would generate an amount of electricity equal to €29,000,000 every year. If the wind farm starts generating and selling power at the beginning of 2018, in what year will it have covered the total installation cost?

a. 2034
b. 2035
c. 2036
d. 2037
e. 2038

Solution: If the wind farm generates €29,000,000 every year, it will take $\frac{€485{,}000{,}000}{€29{,}000{,}000} = 16.7$ years to cover its installation total cost. This would occur in the year 2034. The correct answer is a.

Assumptions and Givens

Since the units of power are $\frac{\text{energy}}{\text{time}}$, a common unit of energy is the watt-hour (Wh). Watt-hours (often kWh or MWh) are typically used to describe how much energy is being used by an appliance or customer. As its name suggests, a Wh is equivalent to 1 W of power being used for

1 hour. Therefore, energy is equal to power x time: $E = P \times t$. For example, a 40-watt light bulb running for 100 hours would use 4kWh of energy

Commented [MF1]: Will this be confusing? You are missing the explanation that to get kWh you have to divide by 1,000?

Question #20:

Now that it is up and running, the SIROCCO wind farm generates an average of 450,000 MW of power. In the residential areas that SIROCCO services, each home uses an average of 600 MWh of energy every day. If SIROCCO is running 24 hours a day, how many homes could it power every day?

a. 12,000
b. 14,000
c. 16,000
d. 18,000
e. 20,000

Solution: To answer this question we use the given equation $E = P \times t$. Here, P is the power generated by SIROCCO (450,000 MW), and *t* is 24 hours. So, every day SIROCCO generates an amount of energy equal to $E = 450{,}000 \text{ MW} \times 24 \text{ hours} = 10{,}800{,}000 \text{ MWh}$. This amount of energy could power $10{,}800{,}000 \times \frac{1 \text{ home}}{600 \text{ MWh}} = 18{,}000$ homes every day. The correct answer is d.