# TEAMS National Competition
# Middle School Version
# Photometry Solution Manual

**25 Questions**

**"Photometry" Questions**

1. When an upright object is placed between the focal point of a lens and a converging lens, how can the resulting image be described?

**Solution:** We can determine the qualities of the reflected image by simply rearranging the equations and observing the sign of the desired quantities. To begin, we know the lens is converging, so the focal distance is some positive number. The object is placed between the lens and the focus of the lens on the left hand side. By the sign convention, the distance to the object is some positive number as well.

In the Lens Equation, we have

$$\frac{1}{f} = \frac{1}{d_0} + \frac{1}{d_i}$$

Solving for $d_i$ yields

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_0}$$

Since $f > d_0$, then $\frac{1}{f} < \frac{1}{d_0}$. So,

$$d_i = \left(\frac{1}{f} - \frac{1}{d_0}\right)^{-1} < 0$$

Which means that $d_i$ is negative. This means the image is ***to the left*** of the lens and must be ***virtual***. To determine the orientation of the image, use the magnification equation.

$$M = -\frac{d_i}{d_0} = -\left(\frac{\left(\frac{1}{f} - \frac{1}{d_0}\right)^{-1}}{d_0}\right) = -\left(\frac{negative}{positive}\right) = \frac{positive}{positive} > 0$$

This means the magnification is positive; moreover,

$$M = \frac{h_i}{h_0}$$

$$h_i = M h_0$$

Since $h_0$ is positive and $M$ is positive, then $h_i$ is positive. Therefore, the image is ***upright.*** Together, the answer is **b) virtual, upright, to the left.**

2. An image is formed by a convex lens 50 cm to the right of the lens. The object is placed 10 cm to the left of the lens. In this configuration, the focal distance is most nearly…

**Solution:** The givens allow us to say $d_i = 50$ and $d_0 = 10$ since the object and image are in the "proper" place. Then we use the Lens Equation,

$$\frac{1}{f} = \frac{1}{50} + \frac{1}{10} = \frac{6}{50}$$

We can then cross multiply and solve for *f*.

$$f = \frac{50}{6} = 8.333\,\ldots$$

Since *f* is positive 12.5, our answer is **b) 8 cm to the right.**

3. Consider a concave lens where the reflection of an object is 40 cm to the left of the lens with a magnification of 10. Find the distance of the object from the lens.

**Solution:** The givens allow us to say $d_i = -7.5$ and $M = \frac{3}{4}$ since the image is on the virtual side. The magnification equation can be used directly,

$$M = -\frac{d_i}{d_0}$$

$$\frac{3}{4} = -\frac{-7.5}{d_0}$$

We can then solve for $d_0$ and find $d_0 = 10$. Since the distance to object is negative, it sits to the left of the lens. Therefore, the answer is **a) 10 cm to the left.**

4. Say you have a concave lens with a focal distance of 12 cm and a magnification of -2/3, find the distance of the reflected image.

**Solution:** The givens allow us to say $f = -12$ and $M = -2/3$ since the lens is concave (and diverging). Multiply both sides of the Lens Equation by $d_i$,

$$d_i\left(\frac{1}{f} = \frac{1}{d_0} + \frac{1}{d_i}\right)$$

$$\frac{d_i}{f} = \frac{d_i}{d_0} + \frac{d_i}{d_i}$$

Substitute the magnification in for $\frac{d_i}{d_0}$, note the minus sign.

$$\frac{d_i}{f} = -M + 1$$

Multiply through by *f*,

$$d_i = -Mf + f$$

Now the quantity we want is completely in terms of known values.

$$d_i = -\left(-\frac{2}{3}\right)(-12) + (-12) = -20$$

Since the distance is negative, the image is virtual and to the left; therefore, the solution is **c) 20 cm to the left.**

5. A convex lens has a focal length of 30 cm. Where is the image of an object that is placed 30 cm left of the lens?

**Solution:** The givens allow us to say $f = 30$ and $d_0 = 30$ since the lens is convex (and converging) and the object is placed to the right. Use the Len's Equation,

$$\frac{1}{f} = \frac{1}{d_0} + \frac{1}{d_i}$$

$$\frac{1}{30} = \frac{1}{30} + \frac{1}{d_i}$$

Attempting to solve for $d_i$ yields the following expression,

$$\frac{1}{d_i} = 0$$

No real number will satisfy this equality, unless we settle on the idea of infinity in the denominator. Since $d_i = \infty$, **e) the image is too far away to be seen.**

6. An object is positioned 5 cm to the left of a converging glass lens which reflects an image 50 cm to the right of the lens. What does this tell you about the image itself in terms of size and orientation?

**Solution:** The givens allow us to say $d_0 = 5$ and $d_i = 50$ since the objects are in their "proper" place. Use the magnification equation,

$$M = -\frac{d_i}{d_0} = \frac{h_i}{h_0}$$

Substitute in known values,

$$-\frac{50}{5} = \frac{h_i}{h_0}$$

Then form the direct variation between $h_0$ and $h_i$ as $h_i = kh_0$ where *k* will be determined.

$$h_i = -10h_0$$

The value of *k* tells us everything we need to know. Since the sign of *k* is negative and *k* is a fraction, we can deduce the image is **d) enlarged and inverted.**

7. An object is positioned 5 cm to the right of a converging glass lens which reflects an image 5 cm from the left of the lens. What does this tell you about the size of the image?

The givens allow us to say $d_0 = -5$ and $d_i = -5$ since the object is placed to the right. Use the magnification equation,

$$M = -\frac{d_i}{d_0} = \frac{h_i}{h_0}$$

Substitute in known values,

$$-\frac{-5}{-5} = \frac{h_i}{h_0}$$

Then form the direct variation between $h_0$ and $h_i$ as $h_i = kh_0$ where *k* will be determined.

$$h_i = -h_0$$

The value of *k* tells us everything we need to know again. Since *k* is -1, we can deduce the size of the image is **c) unchanged.**

8. Suppose we made a lens out of crown class. What is the Coddington shape factor if we have an object 5 cm away being reflected as an image 4 inches away from the lens on the other side?

**Solution:** First we need to convert the 4 inches to centimeters,

$$4\text{ in}\left(\frac{2.54\text{ cm}}{1\text{ in}}\right) = 10.16\text{ cm}$$

Then use the Coddington shape factor equation - note we used crown glass, so *n* = 1.52.

$$C = -\frac{2(n^2-1)}{n+2}\left(\frac{d_i + d_0}{d_i - d_0}\right) = -\frac{2((1.52)^2-1)}{(1.52)+2}\left(\frac{(10.16)+(5)}{(10.16)-(5)}\right) = -2.19$$

This means the answer is **b) -2.19.**

9. If we have a lens which creates a reflected image 30 cm away on one side of an object 10 cm away on the other side, which material produces the Coddington shape factor with the largest magnitude?

**Solution:** Begin with the Coddington shape factor equation and substitute in the known values, $d_i = 30$ and $d_0 = 10$. Therefore, we have the following equation:

$$C = -\frac{2(n^2-1)}{n+2}\left(\frac{30+10}{30-10}\right) = -4\frac{(n^2-1)}{n+2}$$

We're looking for the largest *magnitude* of *C*, so a bigger value of *n* will yield a larger value of *C*. Out of the choices, **d) cubic zirconia** will give us the largest value of 3.66.

10. A light beam hits a cube of ice entirely normal to the surface. What is the angle of refraction?

**Solution:** Since the beam is normal to the surface, $\theta_1 = 0°$. Then, use Snell's Law,

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

Substitute in known values,

$$n_1 \sin(0°) = n_2 \sin(\theta_2)$$

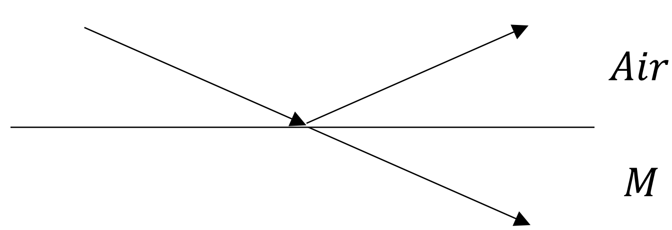
$$0 = n_2 \sin(\theta_2)$$

At this stage, no more work is needed. The only physical value of the angle of refraction that would make sense is **a) 0 degrees.** If not convinced, the last step is calculating

$$\sin^{-1}(0) = \theta_2$$

Which, of course, is 0 degrees.

11. What material *M* best imitates the situation pictured in the following figure.?




**Solution:** By interpreting the figure, we can see that the angle of refraction is the same as the angle of incidence. Since the index of refraction for air is about 1, we can use Snell's Law to figure out what material we need.

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

Since $\theta_1 = \theta_2$ and $n_1 = 1$, then we have

$$n_2 = 1$$

So, we must chose a material with an index of refraction close to 1. Of the options, **e) ice** would approximate the scenario the best since it has the closest index of refraction to 1.

12. A beam of light travels through a slab of crown glass and enters a second slab of crown glass with an angle of incidence of 26 degrees. What is the angle of refraction?

**Solution:** We have beam of light traveling between two slabs of the same material; therefore, the slabs have the same index of refraction ($n_1 = n_2$)

$$n_1 \sin(\theta_1) = n_1 \sin(\theta_2)$$

Then,

$$\sin(\theta_1) = \sin(\theta_2)$$

Take the inverse sine of both sides,

$$\theta_1 = \theta_2$$

Since the angle of incidence is 26 degrees, then the angle of refraction must be **b) 26 degrees** as well.

13. A beam of light traveling down and to the right through a block of Quartz enters a block of Diamond at an angle of 30 degrees clockwise measured from the Diamond to the beam. Find the angle of refraction the refracted beam makes with the normal.

**Solution:** From the table, we see that Quartz has an index of refraction of 1.458 and Diamond has an index of refraction of 2.419. The beam is incident at 30 degrees from the surface, which means the angle of incidence must be 60 degrees from the normal. Use Snell's Law,

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

Substitute in known values,

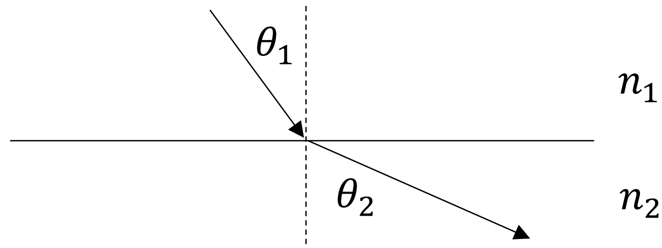
$$(1.458) \sin(60°) = (2.419) \sin(\theta_2)$$

Solve for $\theta_2$

$$\theta_2 = \sin^{-1}\left(\frac{1.458}{2.419} \sin(60°)\right) = 31.47°$$

Therefore the angle of refraction is **b) 31.47 degrees.**

14. Given the following configuration where $\theta_1 < \theta_2$, a beam of light traveling between two mediums, which mathematical relationship *must* be true?




**Solution:** This problem can be solved by testing different examples, but can also be done symbolically. Begin with the given,

$$\theta_1 < \theta_2$$

Take the sine of both sides,

$$\sin(\theta_1) < \sin(\theta_2)$$

We know this is valid since $0° \leq \theta_1, \theta_2 \leq 90°$. Divide both sides by $\sin(\theta_1)$.

$$1 < \frac{\sin(\theta_2)}{\sin(\theta_1)}$$

Go to Snell's Law and find an equivalent expression for $\frac{\sin(\theta_2)}{\sin(\theta_1)}$ by dividing both sides by $n_2 \sin(\theta_1)$.

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

$$\frac{\sin(\theta_2)}{\sin(\theta_1)} = \frac{n_1}{n_2}$$

Substitute this new expression back into the inequality,

$$1 < \frac{n_1}{n_2}$$

Multiply both sides by $n_2$.

$$n_2 < n_1$$

Which means the answer must be **b)** $\boldsymbol{n_1 > n_2}$

15. A controlled beam of light is initially incident on a slab of quartz 30 degrees from the normal, but is adjusted to produce a refracted ray within cubic zirconia at an angle 21.16 degrees from the normal. Approximately what amount of offset in the angle is needed at the point of incidence in order to produce the refracted ray?

**Solution:** From Table 1, we know the index of refraction for quartz is 1.458 and the index of refraction for cubic zirconia is 2.2. We are given an initial angle of incidence $\theta_1 = 30°$ with an unknown offset *x* to result in an angle of refraction $\theta_2 = 21.16°$. With the offset, our true angle of incidence is $\theta_1 = \frac{\pi}{6} + x$ in radians (algebra with degrees won't be pretty). Use Snell's Law,

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

$$(1.458) \sin\left(\frac{\pi}{6} + x\right) = (2.2) \sin(21.16°)$$

Isolate the $\sin\left(\frac{\pi}{6} + x\right)$ term,

$$\sin\left(\frac{\pi}{6} + x\right) = \frac{2.2 \sin(21.16°)}{1.458}$$

$$\sin\left(\frac{\pi}{6} + x\right) = 0.5447$$

Take the inverse sine of both sides,

$$\frac{\pi}{6} + x = \sin^{-1}(0.5447)$$

Solve for *x*,

$$x = \sin^{-1}(0.5447) - \frac{\pi}{6} \approx 0.0524$$

Convert back to degrees,

$$x = 0.0524\left(\frac{180°}{\pi}\right) \approx 3°$$

Therefore the solution is **d) 3 degrees.**

16. What do the sequence of values for the angle of refraction tell you about the change in angular position of the laser beam?

**Solution:** Since the angle of refraction is becoming larger, **b) the beam is gradually moving away from the normal**. The remaining options are not physically true.

17. From the experiment, we can find that $n_1 \propto n_2$ ($n_1$ is proportional to $n_2$). This means there is a number *k* that satisfies the equation $n_1 = kn_2$. Find *k*.

**Solution:** To find *k*, we can rearrange Snell's Law.

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

$$n_1 = \frac{\sin(\theta_2)}{\sin(\theta_1)} n_2$$

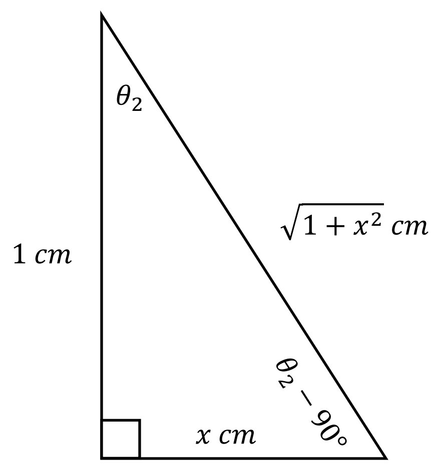
Pick any of the pairs from the data table and calculate *k*

$$k = \frac{\sin(\theta_2)}{\sin(\theta_1)} = \frac{\sin(8.7395°)}{\sin(10°)} \approx 0.875$$

Therefore the constant is **a) 0.875.**

18. At what distance from the reference line does the refracted ray within the second material hit the third material?

**Solution:** We are given that the angle of incidence is 30 degrees, so $\theta_1 = 30°$, with an index of refraction $n_1 = 1$. The second material has an index of refraction $n_2 = 1.5$. The refracted ray forms the hypotenuse for a right triangle in the second material. We know the ray begins 0.5 cm away from the reference, then the second ray intersects the third material *x* cm over; therefore, the refracted ray intersects the third material 0.5 + *x* cm away from the reference.




We can find the value we don't know using the tangent function,

$$\tan(\theta_2) = \frac{x}{1}$$

$$x = \tan(\theta_2)$$

Now we just need to find $\theta_2$, which can be found using Snell's Law.

$$(1)\sin(30°) = (1.5)\sin(\theta_2)$$

$$\theta_2 = \sin^{-1}\left(\frac{\sin(30°)}{1.5}\right)$$

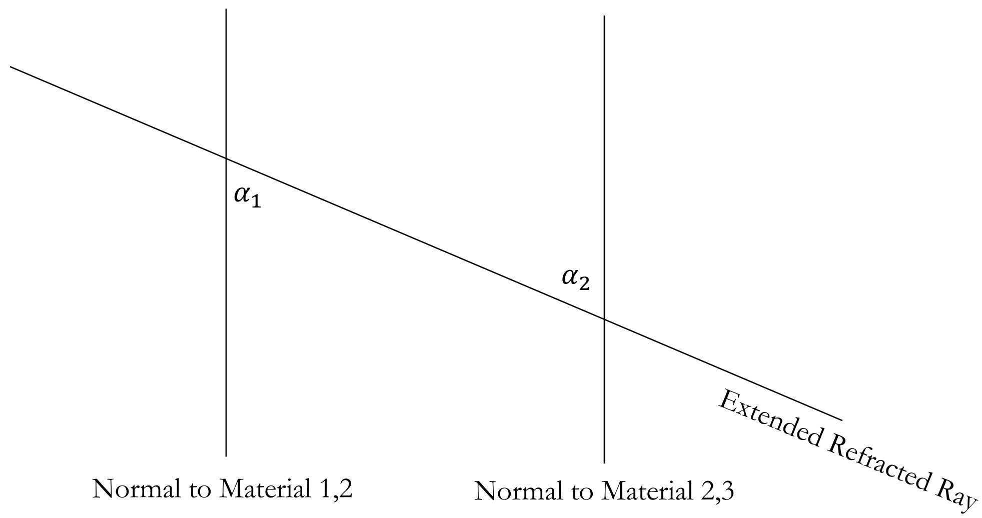
Therefore,

$$x = \tan\left(\sin^{-1}\left(\frac{\sin(30°)}{1.5}\right)\right) \approx 0.35$$

Thus, the desired distance is **b) 0.85 cm.**

19. What is the relationship between the angles $\alpha_1$ and $\alpha_2$?

**Solution:** This problem can be solved through examples, but it can also be done using Euclidian Geometry. We can use geometrical reasoning; only consider the refracted ray and the normals to the second and third materials, extend the refracted ray to ensure it passes through both normal.




No other lines are needed. Since the materials are rectangular slabs as pictured, it is reasonable to assume the surfaces are parallel to one another; moreover, the normals to each surface are parallel as well since the normal is defined to be perpendicular to the surface. Geometrically, we can think of the construction of the normals as a rotation of the surface lines by 90 degrees in either direction. In the Euclidian sense, a rotation is an isometry; therefore, the properties from the previous picture still hold for the new picture – including parallelism.

Next, we are allowed to extend the ray in either direction by Euclid's second postulate: "Any straight line segment can be extended indefinitely in a straight line." The extended refracted ray is then called a *transversal* since it passes through a pair of parallel lines. Provided with this configuration, $\alpha_1$ and $\alpha_2$ can be identified as alternate interior angles. Since $\alpha_1$ and $\alpha_2$ are alternate interior angles, we know that they are congruent. By the definition of congruence, $\alpha_1$ and $\alpha_2$ must have the same angle measure; therefore, the correct relationship is **c)** $\boldsymbol{\alpha_1 = \alpha_2}$**.**

20. What is the angle of refraction as the laser beam enters the third material?

**Solution:** We already solved the problem to an extent earlier, but we will redo the calculations. Begin by finding the necessary angle of incidence from the second material to the third, which can be found using Snell's Law.

$$(1)\sin(30°) = (1.5)\sin(\theta_2)$$

$$\theta_2 = \sin^{-1}\left(\frac{\sin(30°)}{1.5}\right)$$

We know the angle of incidence will be the same as the angle of refraction from the previous material; therefore, we need the next angle $\theta_3$ – which is the angle of refraction we want to find.

$$(1.5)\sin(\theta_2) = (2)\sin(\theta_3)$$

$$\theta_3 = \sin^{-1}\left(\frac{1.5\sin(\theta_2)}{2}\right) = \sin^{-1}\left(\frac{1.5\sin\left(\sin^{-1}\left(\frac{\sin(30°)}{1.5}\right)\right)}{2}\right) = 14.48°$$

Therefore the solution is **d) 14.48 degrees**.

21. What is the angular resolution for a telescope with a lens aperture radius of 1 inch to view red light for the largest wavelength?

**Solution:** Use Rayleigh's Criterion,

$$\theta = 1.22\frac{\lambda}{D}$$

We want to view red light for the largest wavelength, so $\lambda = 750\ nm$. We were given the aperture radius, not diameter, so $D = 2\ in$. We need the diameter in meters, so we convert in to cm.

$$2 \text{ inches}\left(\frac{2.54 \text{ cm}}{1 \text{ inch}}\right) = 5.08 \text{ cm}$$

Then we substitute in the known values into the Rayleigh Criterion,

$$\theta = 1.22\frac{(750 \text{ nm})}{(5.08 \text{ cm})} = 1.801 \text{ x } 10^{-5}$$

This means the solution is **a) $\mathbf{1.801 \text{ x } 10^{-5}}$.**

22. Which color of light is associated with the smallest angular resolution?

**Solution:** Use Rayleigh's Criterion,

$$\theta = 1.22\frac{\lambda}{D}$$

To minimize the value of $\theta$, we just need to locate the light with the smallest value of $\lambda$. That way our denominator is as small as possible. Looking at the table of wavelengths (Table 3), we can see **d) blue** light yields the minimum value.

23. Suppose a telescope has an angular resolution of 1.342 x $10^{-5}$ radians. If the aperture diameter is 5 cm, then what is the light we are able to view?

**Solution:** Use Rayleigh's Criterion,

$$\theta = 1.22\frac{\lambda}{D}$$

We are given the value of $\theta$ and $D$ and we need to solve for $\lambda$.

$$1.342 \text{ x } 10^{-5} = 1.22\frac{\lambda}{(5 \text{ x } 10^{-2})}$$
$$\lambda = 5.5 \text{ x } 10^{-7} \text{ m} = 550 \text{ nm}$$

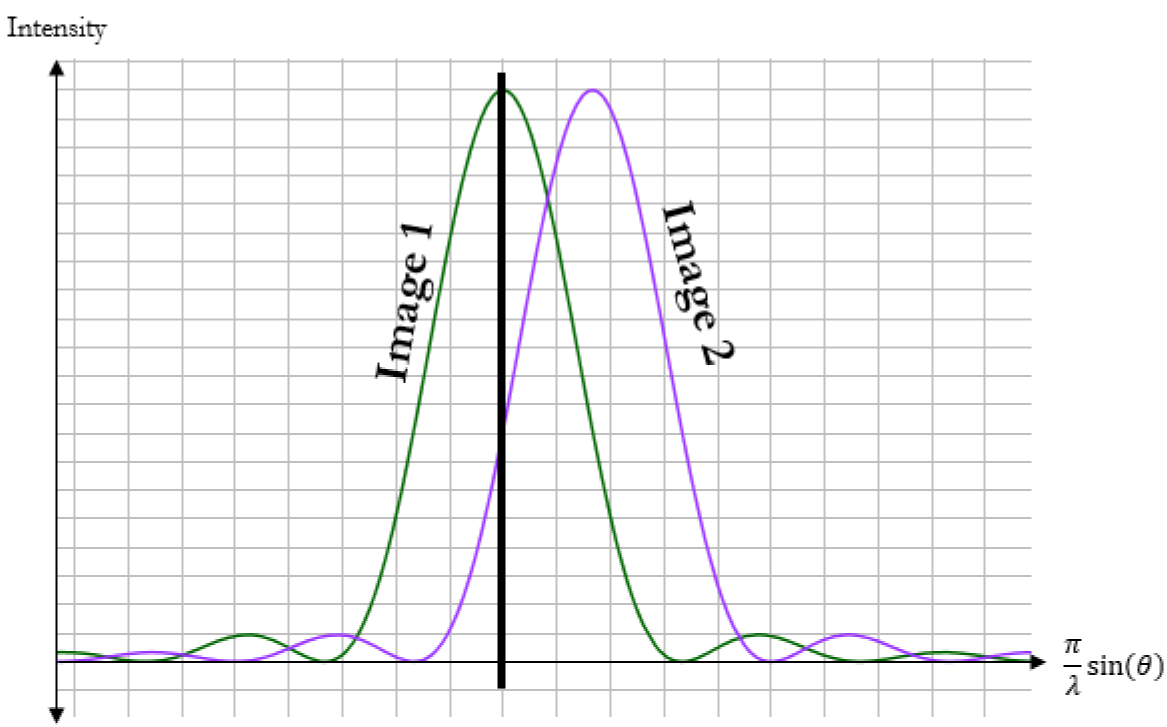
Such a wavelength corresponds to **b) green light.**

24. When is the pattern the darkest for Image 1 or Image 2?

**Solution:** Notice that the intensity is plotted on the *y* axis, meaning the higher the value of the function, the more intense the image. The lack of intensity would mean the image would be dark, this would only happen when **c) either curve reaches a local minimum.**

25. Consider each rectangle width in the plot as 1 unit on the *x* axis. At a minimum, how much does Image 1 be shifted in order to resolve the resulting image?

**Solution:** From the given information, the graphical interpretation of Rayleigh's Criterion is as follows: "In order to distinguish between two patterns, in the worst case, the first minimum of one image must fall at the same point as central maximum of the other. If the patterns overlap too much, then the images cannot be distinguished from one another." In this case, we want to move Image 1. To avoid overlap, we need to move the image to the left, but by how much?



The maximum of the image needs to be at least at the same point as the first minimum of the second image. Since the maximum falls on one of the gridlines, we can easily see that the minimum amount of units we need to move is **b) 2 units to the left.**