Hydropower

Question #1:

A proposed impoundment hydroelectric station is designed for a flow rate of 5 m^3/sec through a penstock and turbine. The difference in water elevation between the reservoir and the river is 60 m. The total efficiency of the dam is 0.75. How much power, in watts, can be realized from this design?

A) 2,207,250 W
B) 220,725,000 W
C) 441,450 W
D) 4,414,500 W
E) 2,943,000 W

Solution: The student must use the power equation. NOTE: No unit conversion is necessary. The efficiency has already been expressed as a decimal.

$$Power = pgZ \times Q \times \varepsilon$$

$$p = 1000 \frac{\text{kg}}{\text{m}^3}$$

$$g = 9.81 \frac{\text{m}}{\text{sec}^2}$$

$$Z = 60 \text{ m}$$

$$\varepsilon = 0.75$$

$$Power = 1000 \frac{\text{kg}}{\text{m}^3} \times 9.81 \frac{\text{m}}{\text{sec}^2} \times 60 \text{ m} \times 5 \frac{\text{m}^3}{\text{sec}} \times 0.75 = 2,207,250 \text{ W}$$

Question #2:

Developers are considering two different impoundment-type hydropower stations with the same operating efficiency, *ε*. The first hydropower station has a proposed static head, *pgZ*, of 100 m and a flow rate, Q_1, of $1000 \text{ m}^3/\text{sec}$. If both plants require the same power output and the second station's static head is 250 m, which of the following equations would you use to solve for the required flow rate, Q_2, of the second power station?

A) $100\text{ m}\left(1000\dfrac{\text{m}^3}{\text{sec}}\right)=250\text{ m}\left(Q_2\right)$

B) $250\text{ m}\left(1000\dfrac{\text{m}^3}{\text{sec}}\right)=100\text{ m}\left(Q_2\right)$

C) $9.81\dfrac{\text{m}}{\text{sec}^2}\times 100\text{ m}\left(1000\dfrac{\text{m}^3}{\text{sec}}\right)=1000\dfrac{\text{kg}}{\text{m}^3}\times 250\text{ m}\left(Q_2\right)$

D) $9.81\dfrac{\text{m}}{\text{sec}^2}\times 250\text{m}\left(1000\dfrac{\text{m}^3}{\text{sec}}\right)=1000\dfrac{\text{kg}}{\text{m}^3}\times 100\text{ m}\left(Q_2\right)$

E) $Z_1\times 100\text{ m}\left(1000\dfrac{\text{m}^3}{\text{sec}}\right)=250\text{ m}\left(Q_2 Z_2\right)$

Solution:

$Power_{plant1} = Power_{plant2}$

$pgZ\times Q_1\times\varepsilon = pgZ\times Q_2\times\varepsilon$

p, g, ε cancel out.

$\text{Z}_1 = 100\text{ m}$

$Q_1 = 1000\dfrac{\text{m}^3}{\text{sec}}$

$Z_2 = 250\text{ m}$

$\text{Z}_1\times Q_1 = Z_2\times Q_2$

$$100\text{ m}\left(1000\dfrac{\text{m}^3}{\text{sec}}\right)=250\text{ m}\left(Q_2\right)$$

Question #3:

Hoover Dam is 220 m tall, has a peak design flow rate of 1060 m^3/sec, and is 85% efficient. Historically, Hoover Dam has only produced 22% of its total potential power. In megawatts, how much less power has the dam produced than its potential?

A) 1516.7

B) 1944.5

C) 2372.3

D) 1484.6

E) 952.7

Solution: Student must use the power equation to calculate potential power. Student must convert efficiency percent as well as the capacity factor percent to decimals. Student must multiply the capacity factor by the potential power to find the actual power. Student must subtract the actual power from the potential power to find the difference.

Potential power = $pgZ \times Q \times \varepsilon$

$$p = 1000 \frac{\text{kg}}{\text{m}^3}$$

$$g = 9.81 \frac{\text{m}}{\text{sec}^2}$$

$$Z = 220 \text{ m}$$

$$\varepsilon = 0.85$$

$$\text{Potential power} = 1000 \frac{\text{kg}}{\text{m}^3} \times 9.81 \frac{\text{m}}{\text{sec}^2} \times 220 \text{ m} \times 1060 \frac{\text{m}^3}{\text{sec}} \times 0.85 = 1{,}944{,}538{,}200 \text{ W}$$

$$1{,}944{,}538{,}200 \text{ W} \times \frac{1 \text{ MW}}{1 \times 10^6 \text{ W}} = 1944.5382 \text{ MW}$$

Actual power = 22% of potential power

Actual power = 0.22 $\times$ 1944.5382 = 427.798404 MW

Difference between potential power and actual power is

$1944.5382 - 427.798404 = 1516.739796$ or about 1516.7 MW.

Question #4:

If power production at Hoover Dam does fall to 1,120 MW as predicted for 2016, and the water flow rate through the system is 800 m^3/sec, with 75% total efficiency, what would be the elevation difference (in meters) between Lake Mead and the Colorado River?

A) 190 D) 1,502

B) 328 E) 570

C) 168

Solution: First, change 1,120 MW (megawatts) to W (watts) using the conversion equation in the givens.

1 MW = 1x10^6 W

$$1{,}120\times\ 1\times10^{6}$$
$$=1{,}120\times1{,}000{,}000$$
$$=1{,}120{,}000{,}000\text{ W}$$

Next, use the formula for power provided in the background information.

$$Power = pgZ\times Q\times\varepsilon$$

The question is asking for the elevation difference between the surface of Lake Mead (the reservoir) and the Colorado River. Note that in the background information, this was defined as variable *Z* having a unit of meters.

Insert the known values for power, water density p, gravity constant g, water flow rate Q, and efficiency, ε then solve for *Z*.

$$1{,}120{,}000{,}000\text{ W} = 1000\frac{\text{kg}}{\text{m}^3}\times9.81\frac{\text{m}}{\text{sec}^2}\times Z\times800\frac{\text{m}^3}{\text{sec}}\times0.75$$

Replace watts W with its equivalent $\frac{\text{kg}\cdot\text{m}^2}{\text{sec}^3}$.

$$1{,}120{,}000{,}000\ \frac{\text{kg}\cdot\text{m}^2}{\text{sec}^3} = 1000\frac{\text{kg}}{\text{m}^3}\times9.81\frac{\text{m}}{\text{sec}^2}\times Z\times800\frac{\text{m}^3}{\text{sec}}\times0.75$$
$$1{,}120{,}000{,}000\ \frac{\text{kg}\cdot\text{m}^2}{\text{sec}^3} = 5{,}886{,}000\frac{\text{kg}\cdot\text{m}}{\text{sec}^3}\times Z$$
$$\frac{1{,}120{,}000{,}000\frac{\text{kg}\cdot\text{m}^2}{\text{sec}^3}}{5{,}886{,}000\ \frac{\text{kg}\cdot\text{m}}{\text{sec}^3}} = Z$$
$$190.28\text{ m}\ \approx\ Z$$

So, the elevation difference between the water surface of Lake Mead and the Colorado River would be approximately 190 meters.

Question #5:

The Bonneville Dam is a hydropower facility located about forty miles east of Portland, Oregon on the Columbia River. The plant has eighteen generators producing an average power output of 6.6×10^{7} W each. What is the potential power output of these generators in kilowatts? Express your answer in scientific notation.

A) 1.188×10^{6}

B) 3.666×10^{12}

C) 1.188×10^{9}

D) 3.666×10^{7}

E) 3.666×10^{8}

Solution: Each generator produces 6.6×10^{7} watts. Multiply this by 18.

$(6.6\times10^{7}\ \text{W})\times18 = 1{,}188{,}000{,}000\ \text{W}$

Use the conversion factor 1 kW = 1000 W.

$$1{,}188{,}000{,}000\ \text{W} \times \frac{1\ \text{kW}}{1000\ \text{W}} = 1{,}188{,}000\ \text{kW}$$

Since you move the decimal place 6 places to the left,

$1{,}188{,}000$ in scientific notation is 1.188×10^{6}.

Question #6:

A Pelton wheel turbine has been selected for a facility with a high reservoir located at an elevation, *H*, of 750 m. The selected turbine will operate with a specific speed, N_S, of 4. If the wheel turns at a rate, *n*, of 300 rotations per minute, how many horsepower can be expected from the turbine?

A) 53,394
B) 2739
C) 1076
D) 1,299,771
E) 23,107

Solution: Use the formula for specific speed.

$$N_S = n\frac{\sqrt{P}}{H^{\frac{5}{4}}}$$

$$N_S = 4$$

$$n = 300 \text{ rpm}$$

Note that the hydraulic head must be in feet, so convert 750 m to feet using 1 ft = 0.3048 m.

$$750 \text{ m} \times \frac{1 \text{ ft}}{0.3048 \text{ m}} = 2460.629 \text{ ft}$$

$$4 = 300\frac{\sqrt{P}}{2460.629^{\frac{5}{4}}} \qquad \text{Solve for } P.$$

$$4 = 300\frac{\sqrt{P}}{17,330.364}$$

$$4(17,330.364) = 300\sqrt{P}$$

$$\frac{4(17,330.364)}{300} = \sqrt{P} \qquad \text{Square both sides.}$$

$$53,394.05 = P$$

Therefore, the power generated is about 53,394 horsepower.

Question #7:

A hydroelectric facility wishes to utilize a single-turbine generator with a rotation rate of 250 rpm. The elevation of the reservoir is 300 feet, and the proposed available power of the generator is 62,500 horsepower. What is the specific speed and the type of turbine used?

A) 31; Francis
B) 3; Pelton
C) 310; Propeller
D) 13; Francis
E) None of these

Solution:

$$N_S = n\frac{\sqrt{P}}{H^{\frac{5}{4}}}$$

$$n = 250 \text{ rpm}$$

$$H = 300 \text{ ft}$$

$$P = 34,225 \text{ hp}$$

$$N_s = 250\frac{\sqrt{34,225}}{300^{\frac{5}{4}}}$$

$$N_s \approx 31$$

Therefore, the specific speed of the turbine is 31 and this type of turbine is a Francis turbine.

Question #8:

Some experts believe that Hoover Dam would have to cease energy production when Lake Mead reaches a minimum elevation of 1050 feet while others say that number could be lowered to 950 feet. Using a linear model with the Lake Mead elevation data for the years 2000 and 2014 predict the year that the minimum of 1050 feet is reached, and the year that the minimum of 950 is reached.

A) 2017; 2029
B) 2016; 2035
C) 2015; 2019
D) 2018; 2025
E) 2020; 2024

Solution: Use the linear equation $y = mx + b$ (in slope-intercept form) or $(y - y_1) = m(x - x_1)$ in point-slope form. Define x as the number of years after the year 2000 and y as the elevation of Lake Mead.

$(y - y_1) = m(x - x_1)$

First, calculate the slope, *m*.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1080.60 - 1199.97}{14 - 0} \approx -8.53$$

$(y - 1199.97) = -8.53(x - 0)$

$y - 1199.97 = -8.53x$

First, replace *y* with 1050 ft and solve for *x*.

$1050 - 1199.97 = -8.53x$

$17.6 \approx x$

Therefore, in 2017, Lake Mead will have an elevation of 1050 feet.

Next, replace *y* with 950 ft and solve for *x*.

$950 - 1199.97 = -8.53x$

$29.3 \approx x$

Therefore, in 2029, Lake Mead will have an elevation of 950 feet.

Question #9:

It is assumed that Hoover Dam produced its maximum power generation of 2074 MW when Lake Mead was at its highest elevation in 1983. Using a proportion, estimate the number of megawatts of energy that will be produced at Hoover Dam if the elevation of Lake Mead falls to 950 feet. How many kilowatts of energy is this?

A) 1,608 MW; 1,608,000 kW
B) 2,675 MW; 2,675,000 kW
C) 1,523 MW; 15,230 kW
D) 1,879 MW; 18,790 kW
E) 1,411 MW; 14,110,000 kW

Solution: The student must use the table of Lake Mead elevations, referring specifically to the data for the year 1983, during which the lake's elevation reached 1,225.44 feet. The student uses this value along with the assumed maximum power production of 2,074 MW and the projected elevation of 950 feet to set up a proportion. One proportion that will work is shown below.

$$\frac{2074\text{ MW}}{1225.44\text{ ft}} = \frac{x\text{ MW}}{950\text{ ft}} \quad \text{Solve for } x.$$

$$1,970,300 = 1225.44x$$

$$1607.8 \approx x$$

So, the projected power production when Lake Mead's elevation is 950 feet is about 1608 MW.

Use the conversion factors $1\text{ MW} = 1\times10^6\text{ W}$ and $1\text{ kW} = 1000\text{ W}$ to convert 1608 MW to kW.

$$1608\text{ MW} \times \frac{1\times10^6\text{ W}}{\text{MW}} = 1,608,000,000\text{ W}$$

$$1,608,000,000\text{ W} \times \frac{1\text{ kW}}{1000\text{ W}} = 1,608,000\text{ kW}$$

Therefore, the projected power production in kilowatts is 1,608,000 kW.

Question #10:

The Three Gorges Dam (completed in China in 2012) has the highest installed hydroelectric generation capacity in the world. The plant contains thirty-two 700 MW turbines and two small 50 MW turbines. In 2013, the plant generated 9509 MW of energy. What was the plant's operational capacity factor during 2013? NOTE: Capacity factor = actual power output / total potential power output.

A) 0.42
B) 0.10
C) 0.26
D) 0.65
E) 0.52

Solution: Potential power output = 32 generators @ 700 MW each + 2 generators @ 50 MW each

$$\text{Potential power output} = 32 \times 700\ \text{MW} + 2 \times 50\ \text{MW} = 22,500\ \text{MW}$$

Actual power output = 9509 MW

$$\begin{aligned}\text{Capacity factor} &= \frac{\text{actual power output}}{\text{potential power output}} \\ &= \frac{9509\ \text{MW}}{22,500\ \text{MW}} \\ &\approx 0.42\end{aligned}$$